Analysis of Engineering Students' Understanding in Differentiate Derivative and Integral

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Abstract. Derivative and integral are such fundamental concepts for Engineering students for their advanced coursed. These two concepts are opposite each other, since Integral as an antiderivative of a function $f(x)$ is a function, whose derivative is equal to $f(x)$. However the Engineering students still confused to identify the difference between derivative and integral. This study aimed to examine how students distinguish the understanding between derivative and integral in a sample of second-year aerospace engineering students in one of the private universities in Yogyakarta, Indonesia, and how they solve it. These two concepts have been taught in their high School and first-year in university. The instrument of this study applied the same problems for two questions, to derived and also to integrated, in order to find students understanding in distinguishing the concepts. Qualitative research was chosen as it can describe the students thinking in answering the test. The result reveals how students differentiate solving Derivative and Integral. One-third of these students do not use the symbol of derivative or integral in solving the question. Most of them just use one symbol, which the majority is derivative, the rest do not use any symbol. Using symbol helps students to answer correctly when dealing with two opposite concepts such as derivative and integral. Thus, conceptual and procedural understanding play a further important role. Integral procedures appear to be avoided because of the complex formulas that involve fractions.

1. Introduction

Engineering students learn mathematics in the different point of view with mathematics students. Bingolbali, E, Monaghan, J & Roper, T [1] said that engineering students view mathematics as a tool and prefer to have just the application aspect in their course. Engineering students also demands to be shown why knowledge of mathematics is essential for their future practical work [2]. They want straightly use the formula to solve the problem rather than find out how the formula works or where it comes from. For example, taking some random number to check a particular number satisfies the equation rather than doing some algebraic formula, which often forgotten by them. Therefore, developing symbol conceptions are suggested for the mathematical education of engineering students followed by the procedural conceptions.

In other hands, students seem to lack representational for the concepts associated with symbolization [3]. For engineering students, Calculus plays a significant role in learning mathematical tools and degree completion requirements to study in universities [4]. In Calculus, a mathematics subject taught for engineering students has two opposite basic concepts that have to be mastered for their advanced course [5]. They are derivative and integral (anti-derivative). These concepts are connected each other, the

symbol are different, but the procedural is similar but in the opposite ways. Thus, they tend to swap the formula or to get misconception [4]. In addition, students still encounter difficulties in the learning of derivatives and integrals since they are such two complex mathematics concepts for many undergraduate students [6].

Meanwhile, derivatives and integrals which are two fundamental concepts in Calculus, are very useful in engineering situations since engineering involves rigorous and precise calculations while derivatives and integral do that [7], for instance, studying change in a variable with respect to change in other variables that using derivative. Also, in the situation which needs to sum up a number of small quantities which are similar with the help of integration [7]. Therefore, it is important for students who take engineering and science to be fluent in integral [8]. The concepts of calculus are complex and repeated like derivative, which is used in the substitution method of integration. In all, derivative and integral are such a fundamental concern in the design and practice of the first-year engineering students [9].

Furthermore, most of the students seemed pleased to see some applications of differential and integral [10]. They thought it is mathematician task to know where it comes from, while for engineering they just need how it works. Students are usually less enthusiastic about many of the projects or complain with quite difficult work. They prefer to enjoy using applications or software to solve the problem [10]. However, to reduce the misconceptions, students must be given a clear insight of the terms of derivative and integrations mathematically [4]. They still need practice sheets and concrete examples when learning derivative and integrations. Since the factors that probably cause difficulties by students in learning integral are the lack of training in solving integral questions and the weak understanding of the basic integral [11]. Based on National Centre for Education Statistics, there are three basic mathematical abilities that students have to mastered [12]. They are conceptual understanding, procedural knowledge and problem solving.

Therefore, this study wants to gather data by testing engineering students to solve 5 functions in two concepts, derivative and integral, in order to analyze how they differentiate those two concepts and solve it.

2. Methods

This study used a descriptive qualitative research through questionnaire to collect students' answers in distinguishing derivatives and integrals. The questionnaire does not aim at reveal frequencies, means or other parameter, but more on determine the diversity of answers within the students [13]. There are 5 simple routine functions that must be derived and integrated. In the designing of the test, the generalization of procedural knowledge and conceptual understanding by NCTM [12] was modified along with the aimed to evaluate student understanding in distinguish between derivative and integral. This instrument has construction validity since it measures aspects of thinking based on the theory [14] which are the procedural knowledge will reveal the use of symbol, condition and processes while the conceptual knowledge will address the essence of principals and relation between derivative and integral as shown in table 1.

Table 1. Designed questions for evaluate students understanding about the difference between derivative and integral by NCTM

Questions	Procedural Knowledge			Conceptual Understanding
	Symbol	Condition	Processes	
$y = x$	• issue the use of derivative symbol like $y' or \frac{dy}{dx}$ when being asked to do differentiation	• there is no power stated	• use the basic rule of integral when asked to do integration $\int x^n\,dx = \frac{1}{n+1}\,x^{n+1}$,	• realize that x has power 1
$y = x^3$		• exactly like the basic rule		• follow the rule
$y = x^{-3}$		• the power of the function is negative		• the negative power will affect the

	• issue the use of integral symbol like $\int y\,dx$ when being asked to do integration		$dengan\ n \neq -1$ • use the basic rule of derivative when asked to do differentiation $y = x^n$ so $y' = nx^{n-1}$	denominator of the integral result
$y = x^{\frac{1}{3}}$		• the power of the function is a fraction		• the fraction power will affect the denominator of the integral result
$y = \sqrt{x}$		• There is no power, but square root.		• the square root has to be changed into the power of a half

The problems do not demand any reasoning or double procedure answer. They just need to do single procedure to solve it in their naturally occurring behaviour [15]. The use of one function for two questions, derivative and integral, is aimed to evaluate how the students distinguish the concept between derivative and integral.

There were seventy-eight students in the early second year of Aerospace Engineering students of Sekolah Tinggi Teknologi Adisutjipto who were tested. They were chosen since they have been learned about derivative and integrals in their first year. After taking the test, they were given a written interview to assure their answer as an approach of methodological triangulation [16]. It is used to check whether the students understand the difference between derivative and integral when doing the test. Then the data were analysed descriptively to illustrate the result of differentiation.

3. Result and Discussion

This study reveals how students experience difficulties in determining what concepts to do when solving derivatives or integrals question. Although, the question has stated what concepts to do derivative or integral like in figure 1. The first analysis on how many numbers were answered correctly on each concept is shown in the table 2.

Table 2. The number of questions answer correctly.

Questions	Correct Answer	Incorrect Answer	No Answer	Sum
Derivative Questions	205 (52.56%)	150 (38.46%)	35 (8.97%)	390
Integral Questions	116 (29.74%)	146 (37.44%)	128 (32.82%)	390
Sum	321 (41.15%)	296 (37.95%)	163 (20.90%)	780

From table 2, we can see that these students more capable to solve derivative rather than integral. More than half correct answer has done for derivative question. Incorrect answer has similar percentage between derivative and integral. Student seemed has difficulties in remembering the concept of integral than derivative since 32.82% students prefer not answered integral questions while for derivative just 8.97% students. Meanwhile, the second analysis on how each student answered the questioners, see table 3 below.

Table 3. The number of the way of students' answer.

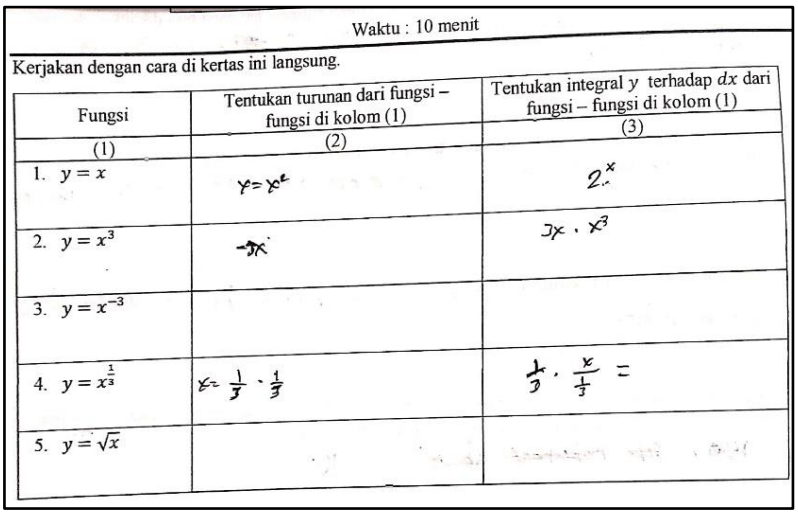
Doing Both Derivative & Integral	Doing Just Derivative	Doing Just Integral
39	38	1
50.00%	48.72%	1.28%
Level of correctness		

Same number of correctness	Derivative correct more	Integral correct more	Zero correct
16	15	5	3
41.03%	38.46%	12.82%	7.69%

Based on table 3, evidently a half of students did both of questions, derivative and integral. Less than half of students did derivative but leave the integral blank. But when we are counting the level of correctness (the number of correct answer), the students who did more of correct answer in derivative rather than integral are 15+38=53 students or 67.94%. There are some reasons that underlie these findings. From the students written interview we can relate and assure reasons behind their answer. Thus based on these analysis, it categorized into three reasons which are;

3.1. No idea at all or trial error

The first reason is students did not have any idea about how to solve the problems. Since there are few students who give impossible answers which has not any relation with the concepts of derivative nor integral instead of leave it blank. For instance, see students' answer in figure 1.



Waktu : 10 menit

Kerjakan dengan cara di kertas ini langsung.

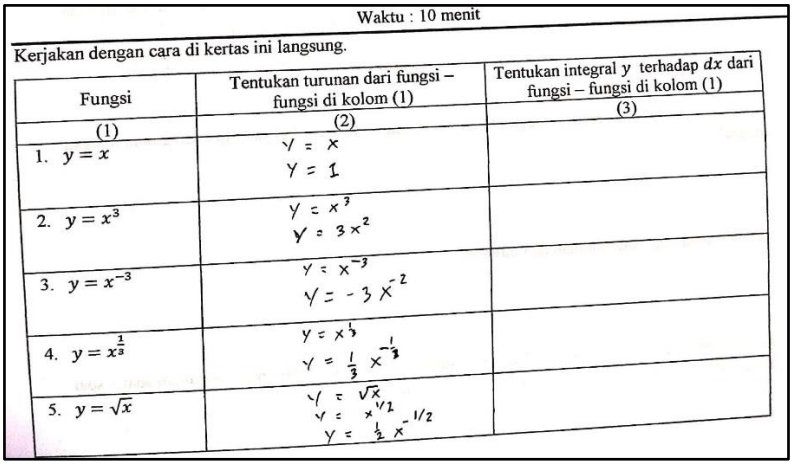
Fungsi	Tentukan turunan dari fungsi – fungsi di kolom (1)	Tentukan integral y terhadap dx dari fungsi – fungsi di kolom (1)
(1)	(2)	(3)
1. $y = x$	$y = x^2$	2^x
2. $y = x^3$	$-3x$	$3x \cdot x^3$
3. $y = x^{-3}$		
4. $y = x^{\frac{1}{3}}$	$\frac{1}{3} \cdot \frac{1}{3}$	$\frac{1}{3} \cdot \frac{x}{\frac{1}{3}} =$
5. $y = \sqrt{x}$		

Figure 1. Student's answer sheet which is all incorrectly.

In the figure 1, we can see that there is no correct answer at all. This student seemed did not get any clue about the concept of derivative or integral. For doing the derivative, it seemed he did not have the basic concept or basic formula by reducing the power. He did not know that at least he just need to locate the power into the beginning of the variable then reduce the number of the power by one. It is worse when he did the integral. We can say that he did not remember any concept about derivative nor integral or do the trial error.

3.2. Good at derivative, bad at integral

The second reasons are students remembered derivative but not integral. Students who are fluent at derivative mostly just did the derivative like the figure 2 below.



Waktu : 10 menit

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Fungsi	Tentukan turunan dari fungsi – fungsi di kolom (1)	Tentukan integral y terhadap dx dari fungsi – fungsi di kolom (1)
(1)	(2)	(3)
1. $y = x$	$y = x$ $y' = 1$	
2. $y = x^3$	$y = x^3$ $y' = 3x^2$	
3. $y = x^{-3}$	$y = x^{-3}$ $y' = -3x^{-2}$	
4. $y = x^{\frac{1}{3}}$	$y = x^{\frac{1}{3}}$ $y' = \frac{1}{3}x^{-\frac{1}{3}}$	
5. $y = \sqrt{x}$	$y = \sqrt{x}$ $y = x^{1/2}$ $y' = \frac{1}{2}x^{-1/2}$	

Figure 2. Good at derivative, leave the integral.

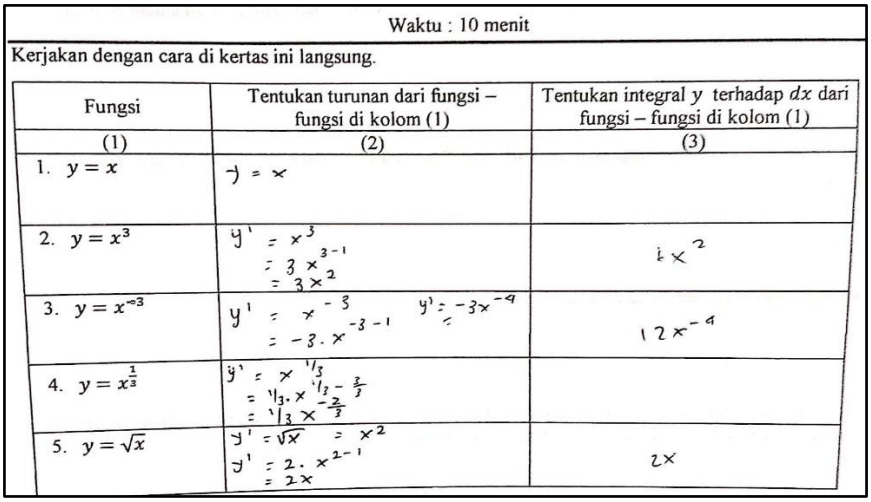
This student in figure 2 just do the derivative questions, they did not solve any single questions of integral. It is interesting since he actually has the concept for derivative but unsuccessful to connect it with integral. The reason is because he needed more time or just did not remember at all. Let us see the interview of him. In the interview, he wrote that he knew the formula of derivative which is $\boldsymbol{if\ y = a^n\ then\ y' = n.a^{n-1}}$. He did not answer questions about integral. When we clarify it he said that he ran out of time to remember about integral.

In order to investigate why students good at derivative but bad at integral, we examined the use of symbol when answering the problems. We got that derivative symbols are better known to students than integrals. The derivative symbols were written by 67.75% of students who only use one symbol (derivative or integral) on their answer sheets.

Table 4. The use of symbol among students.

Using Both Symbol	Using Just One Symbol		No Symbol	Total
	Derivative	Integral		
17	23 67.65%	11 32.35%	27	78
21.79%	43.59%		34.62%	

Based on table 4, just one third of students did not use any symbol when doing derivative and integral questions. Most students have been aware with the symbol of derivative and integral. However, it does not guarantee that they understand the concept as many students were able to use the correct symbol but produce incorrect answer.



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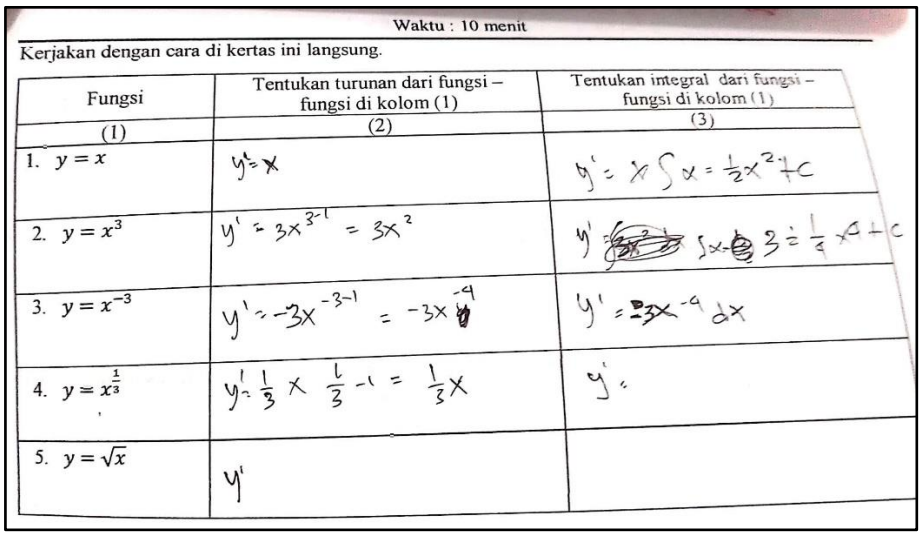
Fungsi	Tentukan turunan dari fungsi – fungsi di kolom (1)	Tentukan integral y terhadap dx dari fungsi – fungsi di kolom (1)
(1)	(2)	(3)
1. $y = x$	$y = x$	
2. $y = x^3$	$y' = x^3$ $= 3x^{3-1}$ $= 3x^2$	$\frac{1}{3}x^2$
3. $y = x^{-3}$	$y' = x^{-3}$ $= -3 \cdot x^{-3-1}$ $y' = -3x^{-4}$	$12x^{-4}$
4. $y = x^{\frac{1}{3}}$	$y' = x^{1/3}$ $= 1/3 \cdot x^{1/3 - 3/3}$ $= 1/3 \, x^{-\frac{2}{3}}$	
5. $y = \sqrt{x}$	$y' = \sqrt{x} = x^2$ $y' = 2 \cdot x^{2-1}$ $= 2x$	$2x$

Figure 3. Lack of integral symbol among students.

From figure 3, we can see that student easily use the symbol of derivative. While for integral, he wrote no symbol, straightly answering which was also incorrect. It means student did not have any idea about integral. Most of students did not use symbol of integral when answering the question as they wrote the final answer. Difference with the derivative questions, students tend to use the symbol *(y')* although sometime they do it incorrectly.

However, there are many wrong answers of integral although they use the correct symbols due to a lack of understanding of the concept as illustrated in figure 4 below. It showed that the student also used symbol of integral but not incomplete. He might be just remembered the symbol $\int \boldsymbol{f(x)}$ without knowing that it will need $\boldsymbol{dx}$. Since the symbol *dx*, called the differential of the variable *x*, its meaning to "with respect to *x*"[17]. Thus we need *dx* to do integration.



Waktu : 10 menit

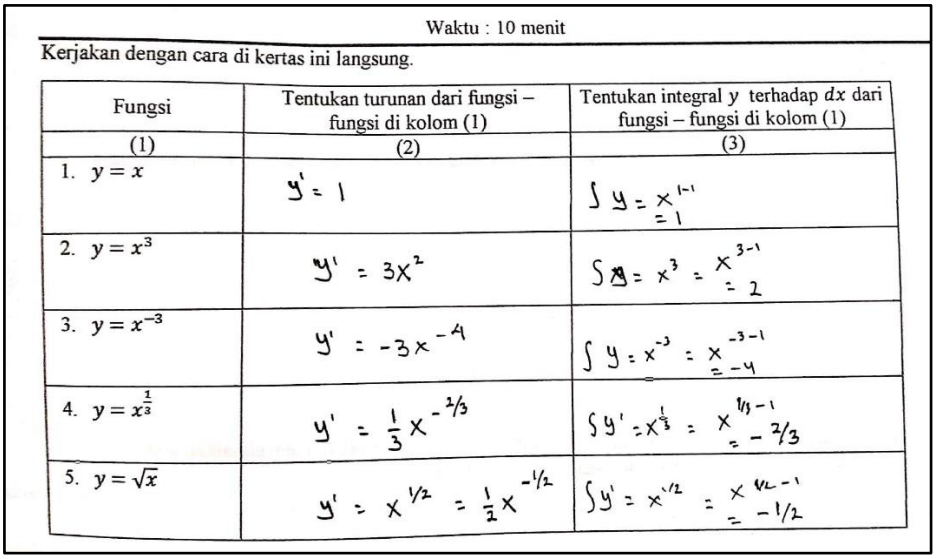
Kerjakan dengan cara di kertas ini langsung.

Fungsi	Tentukan turunan dari fungsi – fungsi di kolom (1)	Tentukan integral dari fungsi – fungsi di kolom (1)
(1)	(2)	(3)
1. $y = x$	$y' = x$	$y' = x \int x = \frac{1}{2}x^2 + C$
2. $y = x^3$	$y' = 3x^{3-1} = 3x^2$	$y' = \int 3x^3 = \frac{1}{4} x^4 + C$
3. $y = x^{-3}$	$y' = -3x^{-3-1} = -3x^{-4}$	$y' = -3x^{-4} dx$
4. $y = x^{\frac{1}{3}}$	$y' = \frac{1}{3} \times \frac{1}{3} - 1 = \frac{1}{3}x$	$y' =$
5. $y = \sqrt{x}$	y'	

Figure 4. Correct symbol but still wrong concepts.

3.3. Good at derivative, bad at integral (wrong of understanding)

Students are still confused about distinguishing big ideas for derivatives and integrals, which is, the derivative will make the power of the variable decreased while the integral will make it increased.



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(1)	(2)	(3)
1. $y = x$	$y' = 1$	$\int y = x^{1-1} = 1$
2. $y = x^3$	$y' = 3x^2$	$\int y = x^3 = x^{3-1} = 2$
3. $y = x^{-3}$	$y' = -3x^{-4}$	$\int y = x^{-3} = x^{-3-1} = -4$
4. $y = x^{\frac{1}{3}}$	$y' = \frac{1}{3}x^{-2/3}$	$\int y' = x^{\frac{1}{3}} = x^{1/3-1} = -2/3$
5. $y = \sqrt{x}$	$y' = x^{1/2} = \frac{1}{2}x^{-1/2}$	$\int y' = x^{1/2} = x^{1/2-1} = -1/2$

Figure 5. Correct symbol but switching concepts.

Furthermore, students were able to do integral but he just did it incorrectly. Figure 5 show students ability in recognizing both symbols, derivative and integral, but failed to remember the concept of integral. He did all derivative questions correctly but not for integral.

3.4. Good at both derivative and integral

There were students who could solve both concepts correctly (see table 3). From a half number of students who did both questions, derivative and integral, 16 students were have the same number of correct answers which 7 out of 16 are perfectly correct. Most of them can solve the three first question which is really simple and easy. As if they could remember the basic of concept of derivative and integral, they were easy to solve the three first questions correctly.

To sum up, it is true that derivative questions are answered correctly rather than integral. In addition, most of students doing derivative rather than integral. In addition, the level of correctness of derivative is bigger than integral. It is caused by lack understanding, switch concept of derivative and integral, wrong understanding, having no understanding at all. Besides, the problem is not only the student' concept understanding but also the teaching learning which less effectively and cognitively [18].

Most of people thinks that learning of algorithms has suffered into two contradictory mind between procedures and understanding, which algorithms have been connected with low-level cognition [18]. But, even if solving procedural problem of derivative and integral consider as the low-level cognition, why most of these students cannot handle it. Algorithms will be best learned by memorization and understanding, since it is linked by "repetition". Meaningful repetition will create deep impression, then lead to memorization, understanding and also procedural fluency [19].

4. Conclusion

Students still does not aware in using the symbol of derivative and integral when solving the problems. However, symbolism is carry and represent the processes of mathematical thinking [20]. Using symbol manipulation can lead to a meaningful conceptual embodiment [21]. By using the symbolism to evoke a process, it can be used to compute the result. By thinking of the symbolism as an object, it can be used as a part of higher level manipulation [20]. While differentiating between the concept are such an area of intellectual activity in mathematics [22]. Some of them even switch those concepts, they reduce the power of variable in doing integral while they add the power of variable when solving derivative. It was our initially hypothesis but instead of finding it we found the interesting finding which is lacking of symbol usage by students. Students more familiar with the symbol of derivative than integral. Students who failed to solve the integral did not use symbol of integral like $\int y\, dx$. On the other hand, almost all students were able to show derivative symbol (y').

Likely, derivative is easier to be solved rather than integral. Most of integral questions are leave it blank by the students. Meanwhile, students need to mastered all mathematical competence including ability to understand the concept and apply the procedural knowledge [23]. Thus, in the future, we need to confirm why students could remember the symbol, the concept and the procedure of derivative rather than integral. It might be related with the teaching experience which need to promotes students' deep learning for their students [24].

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